GK Nichtlinearität und Nichtgleichgewicht in kondensierter Materie

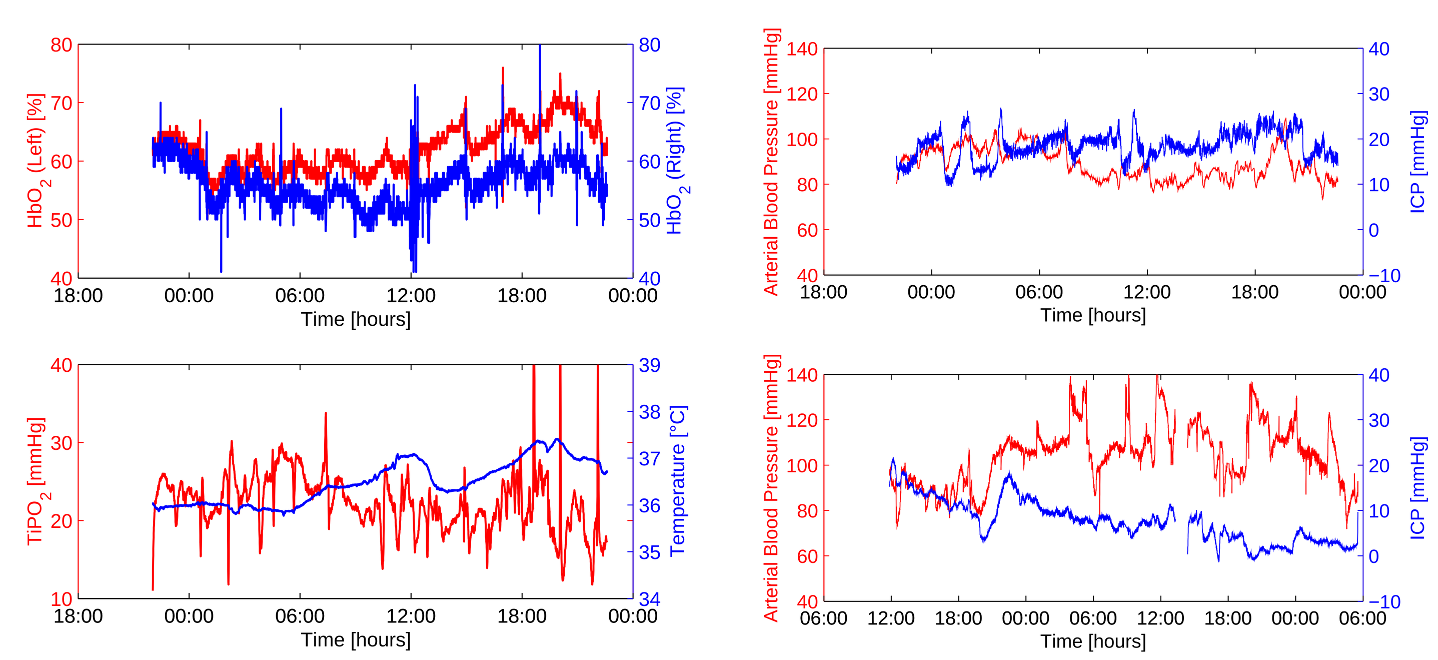
Andreas Jung – A mathematical model of the hydrodynamical processes in the brain

About the Author

- Diploma in May 2000 in Braunschweig at Prof. Weidelt (Geophysics)
- Kollegiat of the Graduiertenkolleg since December 2000
- Activities in the Graduiertenkolleg:
  - Speaker of the Kollegiaten from October 2001 to September 2002
  - Organization of the internal evaluation of the Graduiertenkolleg
  - Organization of projects for the student parabolic flight campaign from the european space agency (ESA). Projects: Granular material and Boomerangs in Zero-G

Introduction & Motivation

Multivariate data from patients with severe head injury on the intensive care unit at the University Hospital Regensburg – Department Neurosurgery.



Goal is the improvement of the treatment. Statistical data analysis can help to:

- understand the data and **reveal** the underlying system
- **determine** the state of health
- if possible, **predict** the future...

With time series analysis and independent component analysis (ICA) one obtains limited results. Solution: **design of a model** for this system.

Cooperations in the GK

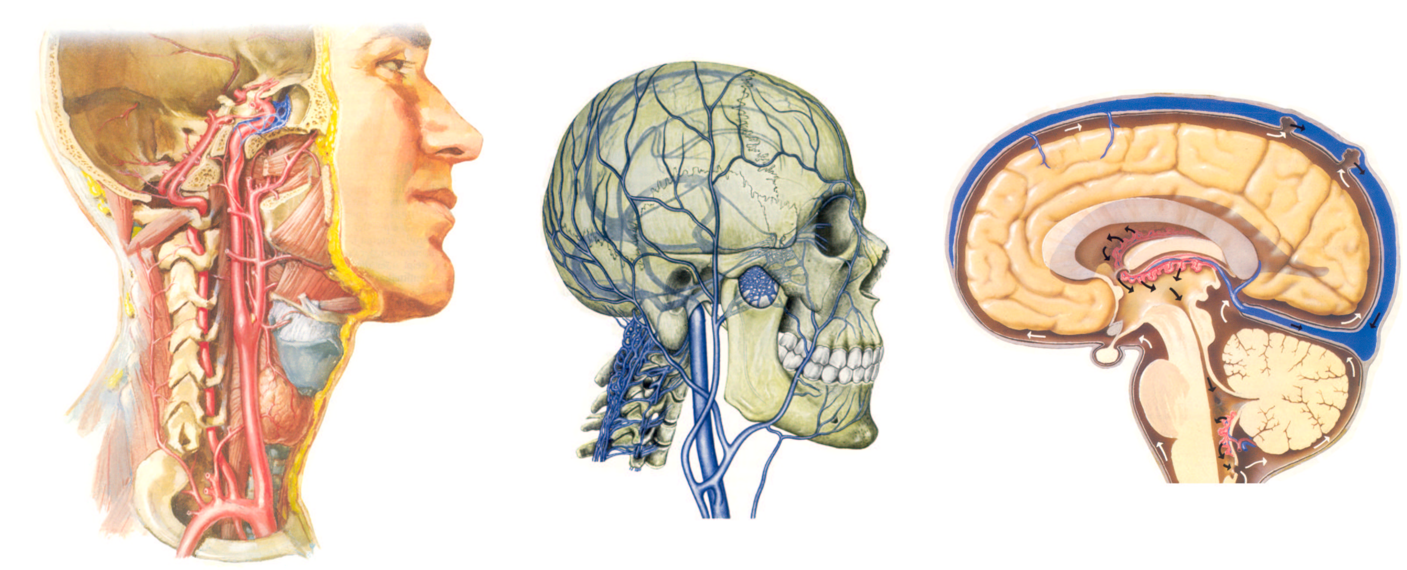
A close cooperation in developing the model has been established with the group of **Prof. Brawanski** of the Department of Neurosurgery at the University Hospital of Regensburg. Especially with Rupert Faltermeier for the physical part of the model and providing the data and Ralf Rothörl, who is a physician at the neurosurgery department.

In the group of **Prof. Richter**, I had very fruitful discussions about the theoretical issues of the model with Juan-Diego Urbina and Peter Schlagheck as well as with Jörg Kaidel from the group of **Prof. Brack**.

Illustrations

Two main fluid circulations exist in the brain:
blood & **cerebrospinal fluid**

Blood supply to the brain via the **arteries**, the outflow of the blood via the **venous blood vessels** and the cerebrospinal fluid (CSF) surrounding the brain tissue – production, circulation and absorption of CSF:

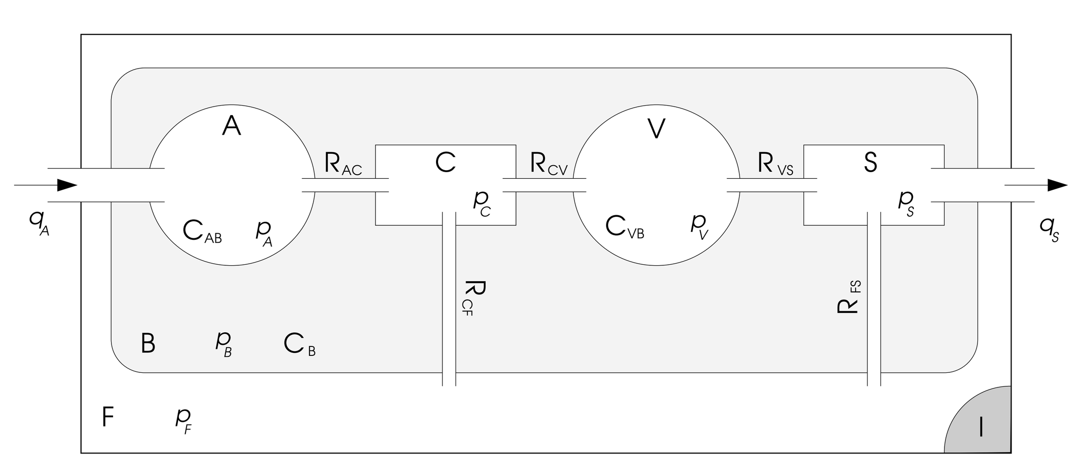


Model & Nonlinear Elements

Using a 7 compartment model:

A=arterial, **C**=capillary, **V**=venous, **S**=sinus,
B=brain tissue, **F**=fluid, **I**=injection of fluid.

A hydrodynamical model of the processes in the brain (an analog electric circuit, which is often more "intuitive" for physicists, can be devolped in the same way).

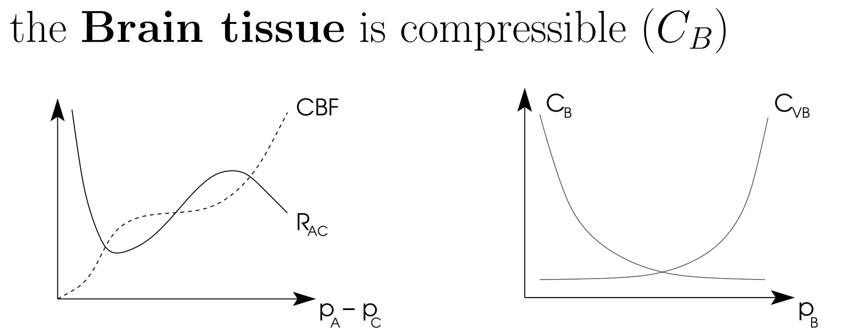


Basic equation: "**conservation of mass**"

$$\sum_i q_i = \frac{dm}{dt} = V\left.\frac{d\rho}{dt}\right|_{V=const} + \rho\left.\frac{dV}{dt}\right|_{\rho=const} \qquad \text{since} \quad m(t) = \rho(t)\cdot V(t)$$

Modeling the nonlinear "Elements":

Autoregulation is a feedback mechanism to ensure constant bloodflow (R_{AC}, C_{AB}), **CSF-Circulation** needs diodes (R_{CF}, R_{FS}), **Veins** have a particular capacity (C_{VB}) and the **Brain tissue** is compressible (C_B)



Differential Equations & Solutions:

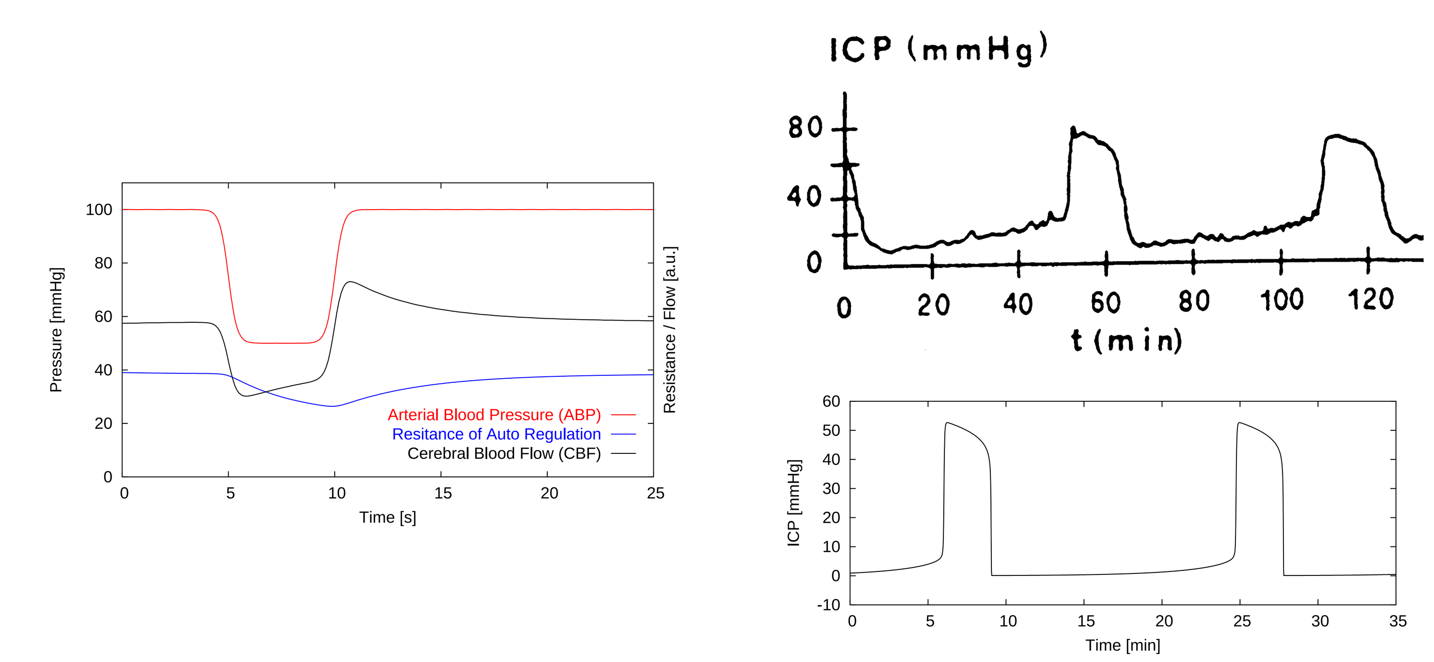
For the simplest model we obtain **two** differential equations plus **one** constrain:

$$\dot{p}_B = \frac{1}{C_{AB}+C_B}\left(\frac{p_C-p_B}{R_{CB}} - \frac{p_B-p_S}{R_{BS}} + q_I + C_{AB}\cdot p_A\right)$$

$$\dot{R}_{AC} = \frac{1}{\tau_{AC}}\left(R_{ACopt}(p_A-p_C) - R_{AC}\right)$$

$$\frac{p_A-p_C}{R_{AC}} - \frac{p_C-p_B}{R_{CB}} - \frac{p_C-p_S}{R_{CS}} = 0$$

The dynamical behaviour of the system (numerical results) shows the following well known clinical phenomena: **Autoregulation** & **ICP plateau waves** (Measurement [Ursino and Lodi, 1997] and Simulation)



Outlook

"**Standard** analysis" of the nonlinear differential equations and their behaviour:

- which numerical solutions do we obtain ?
- do the fix points change to limit cycles, when parameters change ?
- will the system reach chaos ?

"**Stability** analysis": Stability of the fixpoints and their **parameter dependence** – most important for clinical applications!

→ Can we determine the **state of health** of the patient ?

Furthermore, is it possible to...

- **couple** the oxygen-level (Invos and Licox) to the model ?
- can we treat **local behaviour** with this model ?